Propagation of carbon taxes in credit portfolio through macroeconomic factors

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Introduction

We give ourselves a credit portfolio of many companies operating in a country with a closed economy. We would like to know how our portfolio loss will be affected if the regulator decide to introduce sectoral carbon tax in that economy, in order to mitigate the effects of climate change. It is therefore a question of establishing a link between credit risk and climate transition risk.

Many works in the economic and mathematics literature Bangia [1], Nickell [2], Castro [3], Pesaran [4], Baker, Delong, Krugman [5] have been interested in the relationship between economic conditions and default risk or firm valuation. We will draw on these works here.

Plan

A Multisectoral economic model with carbon taxes

Standing assumption 1: the transition scenario

For each sector/representative firm/good $i \in \mathcal{I}$, we introduce deterministic taxes:

- a tax $(\tau_t^i)_{t \geq 0}$ on firm's production,
- a tax $(\zeta_t^{ji})_{t \geq 0}$ on firm i' consumption in sector $j \in \mathcal{I}$,
- and a tax $(\kappa_t^i)_{t \geq 0}$ on household's consumption.

Let $0 \leq t_\circ < t_\star$ be given. The sequences τ, ζ, and κ satisfy

1. for $t \in [0; t_\circ]$, $\mathfrak{d}_t = \mathfrak{d}_0 \in [0,1)^I \times [0,1)^{I \times I} \times [0,1)^I$, namely the taxes are constant;
2. for $t \in (t_\circ, t_\star)$, $\mathfrak{d}_t \in [0,1)^I \times [0,1)^{I \times I} \times [0,1)^I$, the taxes may evolve;
3. for $t \geq t_\star$, $\mathfrak{d}_t = \mathfrak{d}_{t_\star} \in [0,1)^I \times [0,1)^{I \times I} \times [0,1)^I$, namely the taxes are constant.

Main assumption 2: the productivity process

Assumption

We define the $\mathbb{R}^I$-valued process $\mathcal{A}$ which evolves according to

$$\begin{cases} \mathcal{A}_t &= \mathcal{A}_{t-1} + \Theta_t, \\ \Theta_t &= \mu + \Gamma\Theta_{t-1} + \varepsilon\mathcal{E}_t, \end{cases} \quad \textit{for all } \mathbf{t} \in \mathbb{N}^*,$$

with $\mu \in \mathbb{R}^I$, $\mathcal{A}_0 \in \mathbb{R}^I$ and where the matrix $\Gamma \in \mathbb{R}^{I \times I}$ has eigenvalues all strictly less than 1 in absolute value, $0 < \varepsilon \leq 1$ is an intensity of noise parameter that is fixed.

Firms

For $i \in \mathcal{I}$, we consider a representative firm of sector i, with technology described at time $t \in \mathbb{N}$ by the production function

$$Y_t^i = A_t^i (N_t^i)^{\psi^i} \prod_{j \in \mathcal{I}} (Z_t^{ji})^{\lambda^{ji}}, \tag{1}$$

where, for each $t \in \mathbb{N}$ and for $i, j \in \mathcal{I}$,

- $A_t^i := \exp\{\mathcal{A}_t^i\}$ represents the sectoral level of technology;
- N_t^i denotes sectoral hours of work or employment, valued in $\mathbb{R}_+^*$;
- Z_t^{ji} denotes the intermediary input produced by sector j and used in sector i at time t, valued in $\mathbb{R}_+^*$;
- $(\lambda^{ji})_{i,j \in \mathcal{I}} \in (0,1)$ and $(\psi^i)_{i \in \mathcal{I}} \in (0,1)$ are the elasticities and are such that for each $i \in \mathcal{I}$, $\lambda_i + \sum_{j \in \mathcal{I}} \lambda^{ji} = 1$, namely that every sector supplies all the other sectors in the economy.

For each sector i, for each time t, for given P_t^i, W_t^i, A_t^i, τ_t^i, and $(Z_t^{ji})_{j \in \mathcal{I}}$, the firm maximises her profits by solving

$$\max_{N_t^i, Z_t^{1i}, \ldots, Z_t^{Ii}} \left\{ P_t^i (1 - \tau_t^i) Y_t^i - W_t^i N_t^i - \sum_{j \in \mathcal{I}} Z_t^{ji} (1 + \zeta_t^{ji}) P_t^j \right\}, \quad \text{subject to (1).} \tag{2}$$

Households

The representative household consumes the I goods of the economy and provides labor to all the sectors. For any $(C, H) \in \mathscr{A}$, let

$$\mathcal{J}(C, H) := \sum_{i \in \mathcal{I}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U(C_t^i, H_t^i)\right],$$

where the time preference parameter $\beta \in [0, 1)$ and a utility function U is given, for $\varphi \in [0, +\infty)$ and $\sigma \in [0, +\infty)$, by $U(x, y) := \frac{x^{1-\sigma}}{1-\sigma} - \frac{y^{1+\varphi}}{1+\varphi}$ on $(0, \infty)^2$.

The representative household seeks to maximize its objective function by solving

$$\max_{(C,H) \in \mathscr{A}} \quad \mathcal{J}(C, H), \tag{3}$$

where for any $C, H \in \mathscr{L}_+^1(\mathbb{G}, (0, \infty)^I)$, we introduce the wealth process

$$Q_t = (1 + r_{t-1})Q_{t-1} + \sum_{i \in \mathcal{I}} W_t^i H_t^i - \sum_{i \in \mathcal{I}} P_t^i (1 + \kappa_t^i) C_t^i, \qquad \text{for any } t \geq 0, \tag{4}$$

with the convention $Q_{-1} = 0$ and $r_{-1} = 0$.

Market clearing and equilibrium

Definition

We define goods' and labor's market clearing conditions for each sector $i \in \mathcal{I}$, at $t \in \mathbb{N}$, as $Y_t^i = C_t^i + \sum_{j \in \mathcal{I}} Z_t^{ij}$ and $H_t^i = N_t^i$, $\mathbb{P}$-almost surely respectively.

Definition

We define market equilibrium at $t \in \mathbb{N}$ as a market where both firms and households maximize their profits/utilities, i.e. where both (2) and (3) are satisfied.

All the endogenous real variables can be determined by the knowledge of $(Y_t^i)_{i \in \mathcal{I}, t \in \mathbb{N}}$, $(C_t^i)_{i \in \mathcal{I}, t \in \mathbb{N}}$, the productivity/technology, taxes, and the model's parameters. Solving the model consists then in determining Y and C.

For each time $t \in \mathbb{N}$, we obtain a nonlinear system with $2I$ equations and $2I$ variables, therefore computationally heavy to solve for each taxes trajectory and productivity scenario.

Output and consumption dynamics

Theorem

Assume that:

1. $\sigma = 1$,
2. $\mathrm{I}_I - \boldsymbol{\lambda}$ *is not singular,*
3. $\mathrm{I}_I - \Lambda(\mathfrak{d}_t)^\top$ *is not singular for all* $t \geq 0$.

Then there exist unique C_t *and* Y_t *satisfying the problem. Moreover, introducing*

$$\mathfrak{e}_t = \mathfrak{e}(\mathfrak{d}_t) := (\mathrm{I}_I - \Lambda(\mathfrak{d}_t)^\top)^{-1}1, \tag{5}$$

and $\mathcal{B}_t = (\mathcal{B}_t^i)_{i\in\mathcal{I}} := \left[\mathcal{A}_t^i + v^i(\tau_t, \kappa_t, \zeta_t)\right]_{i\in\mathcal{I}}$ *with*

$$v^i(\tau_t, \kappa_t, \zeta_t) = \log\left((\mathfrak{e}_t^i)^{-\frac{\phi\psi^i}{1+\phi}} \left(\Psi^i(\mathfrak{d}_t)\right)^{\frac{\psi^i}{1+\phi}} \prod_{j\in\mathcal{I}} \left(\Lambda^{ji}(\mathfrak{d}_t)\right)^{\boldsymbol{\lambda}^{ji}}\right), \tag{6}$$

we have

$$C_t = \exp\left\{\left((\mathrm{I}_I - \boldsymbol{\lambda})^{-1}\mathcal{B}_t\right)\right\}, \tag{7}$$

where for all $i \in \mathcal{I}$,

$$Y_t^i = \mathfrak{e}_t^i C_t^i. \tag{8}$$

Output growth and consumption growth dynamics

Theorem

For any $t \in \mathbb{N}^*$, *let* $\Delta_t^Y := \log(Y_t) - \log(Y_{t-1})$, $\Delta_t^C := \log(C_t) - \log(C_{t-1})$. *Then*

$$\Delta_t^{\varpi} \sim \mathcal{N}\left(m_t^{\varpi}, \widehat{\Sigma}\right), \qquad \text{for } \varpi \in \{Y, C\}, \tag{9}$$

with

$$\widehat{\Sigma} = \varepsilon^2 (\mathrm{I}_I - \boldsymbol{\lambda})^{-1} \overline{\Sigma} (\mathrm{I}_I - \boldsymbol{\lambda}^\top)^{-1}, \tag{10}$$

$$m_t^C = (I - \boldsymbol{\lambda})^{-1} \left[\overline{\mu} + v(\mathfrak{d}_t) - v(\mathfrak{d}_{t-1})\right], \tag{11}$$

$$(m_t^Y)^i = (m_t^C)^i + \log\left(\mathfrak{e}^i(\mathfrak{d}_t)\right) - \log\left(\mathfrak{e}^i(\mathfrak{d}_{t-1})\right), \quad \text{for all } i \in \mathcal{I}, \tag{12}$$

where $\overline{\mu}$ *and* $\varepsilon^2\overline{\Sigma}$ *are the mean and the variance of the stationary process* Θ *(Assumption 1),* v *is defined in* (6) *and* $\mathfrak{e}$ *in* (5).

Remarks and comments

From now on, we suppose that the taxes κ and τ are calibrated on the carbon price, precisely, for each sector $i \in \mathbb{N}$ and each time $t \in \mathbb{N}$, τ_t^i and κ_t^i are functions of δ_t. It is therefore a model of propagation of the carbon taxes calibrated on the carbon price in a closed economy.

Tnen, from Standing Assumptions 1, our economy follows three regimes:

- Before the climate transition where there is not carbon taxes, the economy is a stationary state leads by the productivity.
- During the transition, the economy is in a transitory state lead by the productivity and the carbon taxes.
- After the transition, we reach a constant carbon price and the economy return in a stationary state rules by the productivity.

A Firm valuation model

Assumptions

Assumption

The $\mathbb{R}^N$-valued return process on assets of the firms denoted $(\omega_t)_{t\in\mathbb{N}^*}$ is linear in the economic factors (consumption growth by sector introduced in Theorem (5)), specifically we set

$$\omega_t = \tilde{\mathfrak{a}}\Delta_t^C + \mathfrak{b}_t, \tag{13}$$

where $\tilde{\mathfrak{a}} \in \mathbb{R}^{N\times I}$, and where the idiosyncratic noise $(\mathfrak{b}_t)_{t\in\mathbb{N}}$ is i.i.d. with law $\mathcal{N}(0, \mathrm{diag}(\sigma^2_{\mathfrak{b}^n}))$ where $\sigma_{\mathfrak{b}^n} > 0$ for all $n \in \{1, \ldots, N\}$. Moreover, $(\Delta_t^C)_{t\in\mathbb{N}^*}$ and $(\mathfrak{b}_t)_{t\in\mathbb{N}}$ are independent.

The above definition of the assets return rewrites, setting $\mathfrak{a} := \tilde{\mathfrak{a}}(\mathrm{I}_I - \boldsymbol{\lambda})^{-1}$

$$\omega_t := \mathfrak{a}\left(\Theta_t + v(\mathfrak{d}_t) - v(\mathfrak{d}_{t-1})\right) + \mathfrak{b}_t. \tag{14}$$

Definition

Definition

Considering the Discounted Cash Flow method, following Kruschwitz and Löffer [6], the firm value is the infinite sum of the present value of the cash flows. For all time $t \geq 0$ and firm $n \in \{1, \ldots, N\}$, we note F_t^n the free cash flows of n at t, and $r > 0$ the discount rate. Then, the value V_t^n of the firm n, at time t, is

$$V_t^n := \mathbb{E}_t \left[\sum_{s=0}^{+\infty} e^{-rs} F_{t+s}^n \right]. \tag{15}$$

An expanded expression of the firm value is

$$V_t^n = F_t^n \left(1 + \sum_{s=1}^{+\infty} e^{-rs} \mathbb{E}_t \left[\exp \left(\mathfrak{a}^{n\cdot} \sum_{u=1}^{s} \Theta_{t+u} + \mathfrak{a}^{n\cdot} \left(v(\mathfrak{d}_{t+s}) - v(\mathfrak{d}_t) \right) + \sum_{u=1}^{s} \mathfrak{b}_{t+u}^n \right) \right] \right).$$

Let us introduce, for a firm n and $t \in \mathbb{N}$, the quantity

$$\mathcal{V}_t^n = F_t^n \left(1 + \sum_{s=1}^{+\infty} e^{-rs} \mathbb{E}_t \left[\exp \left(s \mathfrak{a}^{n\cdot} \overline{\mu} + \mathfrak{a}^{n\cdot} \left(v(\mathfrak{d}_{t+s}) - v(\mathfrak{d}_t) \right) + \sum_{u=1}^{s} \mathfrak{b}_{t+u}^n \right) \right] \right) \tag{16}$$

Firm value

Theorem

For any $n \in \{1, \dots, N\}$, assume that $\varrho_n := \frac{1}{2}\sigma^2_{\mathfrak{b}_n} + \mathfrak{a}^{n\cdot}\overline{\mu} - r < 0$; then $\mathcal{V}^n_t$ is well defined for all $t \in \mathbb{N}$ and

$$\mathcal{V}^n_t = F^n_0 \mathfrak{R}^n_t(\mathfrak{d}) \exp\{(\mathfrak{a}^{n\cdot}(\mathcal{A}^\circ_t - v(\mathfrak{d}_0)))\} \exp\left(\mathcal{W}^n_t\right), \tag{17}$$

for all $t \in \mathbb{N}^*$, with

$$\mathfrak{R}^n_t(\mathfrak{d}) := \sum_{s=0}^{\infty} e^{\varrho_n s} \exp\{(\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t+s}))\}.$$

And explicitly,

$$\mathfrak{R}^n_t(\mathfrak{d}) = \begin{cases} \dfrac{e^{\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t_\star})}}{1 - e^{\varrho_n}}, & \text{if } t \geq t_\star, \\ \displaystyle\sum_{s=0}^{t_\star - t} e^{\varrho_n s} \exp\left\{\left(\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t+s})\right)\right\} + \dfrac{e^{\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t_\star}) + \varrho_n (t_\star - t + 1)}}{1 - e^{\varrho_n}}, & \text{if } t_\circ \leq t < t_\star \\ e^{\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t_\circ})} \dfrac{1 - e^{\varrho_n (t_\circ - t)}}{1 - e^{\varrho_n}} + \displaystyle\sum_{s=t_\circ - t+1}^{t_\star - t} e^{\varrho_n s} \exp\left\{\left(\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t+s})\right)\right\} + \dfrac{e^{\mathfrak{a}^{n\cdot} v(\mathfrak{d}_{t_\star}) + \varrho_n (t_\star - t + 1)}}{1 - e^{\varrho_n}}, & \text{otherwise}. \end{cases} \tag{18}$$

Remarks

1. For $t, T \in \mathbb{N}$ and $1 \le n \le N$, denote

$$\mathcal{K}^n(\mathfrak{d}, t, T, \mathcal{A}_t^\circ, \Theta_t) := \mathfrak{m}^n(\mathfrak{d}, t, \mathcal{A}_t^\circ) + \log\left(\frac{\mathfrak{R}_{t+T}^n(\mathfrak{d})}{\mathfrak{R}_t^n(\mathfrak{d})}\right) + \mathfrak{a}^{n\cdot}\Gamma\Upsilon_{T-1}\Theta_t + \mathfrak{a}^{n\cdot}\left(\sum_{u=1}^{T}\Upsilon_{u-1}\right)\mu, \tag{19}$$

and

$$\mathcal{L}^n(t, T) := \sqrt{\sigma_{\mathfrak{b}^n}^2(t+T) + \varepsilon^2\sum_{u=1}^{T}(\mathfrak{a}^{n\cdot}\Upsilon_{T-u})\Sigma(\mathfrak{a}^{n\cdot}\Upsilon_{T-u})^\top}. \tag{20}$$

We have

$$\log\left(\mathcal{V}_{t+T}^n\right)|\mathcal{G}_t \sim \mathcal{N}\left(\mathcal{K}^n(\mathfrak{d}, t, T, \mathcal{A}_t^\circ, \Theta_t), \mathcal{L}^n(t, T)\right).$$

2. Assume that $|\Gamma| < 1$ and that

$$\rho := \max_{1\le n\le N} \mathfrak{a}^{n\cdot}\overline{\mu} + \frac{1}{2}\sigma_{\mathfrak{b}_n}^2 + \frac{\varepsilon^2}{2}|\mathfrak{a}^{n\cdot}|^2|\sqrt{\Sigma}|^2(1-|\Gamma|)^{-2} < r\,. \tag{21}$$

then

$$\mathbb{E}\left[|V_t^n - \mathcal{V}_t^n|\right] \le C\varepsilon, \tag{22}$$

for some positive constant C (depending on t, ρ).

A Credit risk model

Definitions and assumptions

Assumption

Consider a portfolio of $N \in \mathbb{N}^*$ credits. For all $1 \leq n \leq N$,

1. $(\mathrm{EAD}_t^n)_{t \in \mathbb{N}^*}$ *is a $\mathbb{R}_*^+$-valued deterministic process.*
2. $(\mathrm{LGD}_t^n)_{t \in \mathbb{N}^*}$ *is a $(0, 1]$-valued deterministic process.*
3. B^n *is a deterministic scalar.*

Definition

For $t \geq 1$, the potential loss of the portfolio at time t is defined as

$$L_t^N := \sum_{n=1}^{N} \mathrm{EAD}_t^n \cdot \mathrm{LGD}_t^n \cdot 1_{\{\mathcal{V}_t^n \leq B^n\}}. \tag{23}$$

The losses

The following theorem, similar to the one introduced in [7, Propositions 1, 2] and used when a portfolio is perfectly fine grained, shows that we can approximate the portfolio loss by the conditional expectation of losses given the systemic factor.

Theorem

For all $t \in \mathbb{N}$, define

$$\mathrm{L}_t^{\mathbb{G},N} := \mathbb{E}\left[L_t^N \middle| \mathcal{G}_t\right] = \sum_{n=1}^{N} \mathrm{EAD}_t^n \cdot \mathrm{LGD}_t^n \cdot \Phi\left(\frac{\log B^n - \mathfrak{m}^n(\mathfrak{d}, t, \mathcal{A}_t^\circ)}{\sigma_{\mathfrak{b}^n}\sqrt{t}}\right), \tag{24}$$

where $\mathfrak{m}^n(\mathfrak{d}, t, \mathcal{A}_t^\circ)$ is defined in Corollary 17. Under some assumptions, $L_t^N - \mathrm{L}_t^{\mathbb{G},N}$ converges to zero almost surely as N tends to infinity, for all $t \in \mathbb{N}$.

Assumption

Definition

Let $t \geq 0$ be the time when the risk measure is computed for a period $T \geq 1$. As classically done, the potential loss is separated into three components [8]:

- The (conditional) Expected Loss (EL) is the amount that an institution expects to lose on a credit exposure seen at t and over a given time horizon T.

$$\mathrm{EL}_t^{N,T} := \mathbb{E}\left[\mathrm{L}_{t+T}^{\mathbb{G},N} \middle| \mathcal{G}_t\right]. \tag{25}$$

- The Unexpected Loss (UL) is the amount by which potential credit losses might exceed EL. For $\alpha \in (0,1)$,

$$\mathrm{UL}_t^{N,T}(\alpha) := \mathrm{VaR}_t^{\alpha,N,T} - \mathrm{EL}_t^{N,T}, \quad \text{where} \quad 1-\alpha = \mathbb{P}\left[L_{t+T}^{\mathbb{G},N} \leq \mathrm{VaR}_t^{\alpha,N,T} \middle| \mathcal{G}_t\right]. \tag{26}$$

- The Stressed Loss (or Expected Shortfall or ES) is the amount by which potential credit losses might exceed the capital requirement $\mathrm{VaR}_t^{\alpha}(L_s^N)$:

$$\mathrm{ES}_t^{N,T}(\alpha) := \mathbb{E}\left[L_{t+T}^N \middle| L_{t+T}^N \geq \mathrm{VaR}_t^{\alpha,N,T}, \mathcal{G}_t\right], \qquad \text{for } \alpha \in (0,1). \tag{27}$$

Probability of Default & Expected and Unexpected Losses

Proposition

Let $(\Upsilon_u := \sum_{s=0}^{u} \Gamma^s)_{u\in\mathbb{N}}$ *and* $(\mathfrak{R}_u^n(\mathfrak{d}))_{u\in\mathbb{N}}$ *(with* $n \in \{1,\dots,N\}$*) be as in* (8)*. For* $(a,\theta) \in \mathbb{R}^I \times \mathbb{R}^I$*,* $t \in \mathbb{N}$*,* $T \in \mathbb{N}^*$*, and* $n \in \{1,\dots,N\}$*, define*

$$\mathfrak{L}^n(\mathfrak{d},t,T,a,\theta) := \Phi\left(\frac{\log B^n - \mathcal{K}^n(\mathfrak{d},t,T,a,\theta)}{\mathcal{L}^n(t,T)}\right),$$

where $\mathcal{K}^n(\mathfrak{d},t,T,a,\theta)$ *and* $\mathcal{L}^n(t,T)$ *are defined in Remark 17. Then, the (conditional) probability of default of the entity* n *at time* t *over the horizon* T *is*

$$\mathrm{PD}^n_{t,T,\mathfrak{d}} := \mathbb{P}\left(\mathcal{V}^n_{t+T} \leq B^n | \mathcal{G}_t\right) = \mathfrak{L}^n(\mathfrak{d},t,T,\mathcal{A}_t^\circ,\Theta_t), \tag{28}$$

and the (conditional) EL of the portfolio at time t *over the horizon* T *reads*

$$\mathrm{EL}^{N,T}_{t,\mathfrak{d}} := \mathrm{EL}^{N,T}_t = \sum_{n=1}^{N} \mathrm{EAD}^n_{t+T} \cdot \mathrm{LGD}^n_{t+T} \cdot \mathfrak{L}^n(\mathfrak{d},t,T,\mathcal{A}_t^\circ,\Theta_t). \tag{29}$$

Carbon price sensitity

Definition

For our portfolio of N firms and for $\alpha \in (0,1)$, we introduce the sensitivity of expected and unexpected losses to carbon taxes, respectively denoted $\Gamma^{N,T,\mathrm{EL}}_{t,\mathfrak{d}}(\mathfrak{U})$ and $\Gamma^{N,T,\mathrm{UL}}_{t,\mathfrak{d},\alpha}(\mathfrak{U})$, as being, for a given sequence of carbon taxes $\mathfrak{d}$,

$$\Gamma^{N,T,\mathrm{EL}}_{t,\mathfrak{d}}(\mathfrak{U}) := \lim_{\vartheta \to 0} \frac{\mathrm{EL}^{N,T}_{t,\mathfrak{d}+\vartheta\mathfrak{U}} - \mathrm{EL}^{N,T}_{t,\mathfrak{d}}}{\vartheta}, \tag{30}$$

and

$$\Gamma^{N,T,\mathrm{UL}}_{t,\mathfrak{d},\alpha}(\mathfrak{U}) := \lim_{\vartheta \to 0} \frac{\mathrm{UL}^{N,T}_{t,\mathfrak{d}+\vartheta\mathfrak{U},\alpha} - \mathrm{UL}^{N,T}_{t,\mathfrak{d},\alpha}}{\vartheta}, \tag{31}$$

where $\mathfrak{U} \in ([0,1)^I \times [0,1)^{I\times I} \times [0,1)^I)^{\mathbb{N}}$ is chosen so that there exists a neighbourhood v of 0 so that for all $\vartheta \in v$, $\mathfrak{d} + \vartheta\mathfrak{U} \in ([0,1)^I \times [0,1)^{I\times I} \times [0,1)^I)^{\mathbb{N}}$.

Simulations

Scenarios and Sectors

The scenarios used come from the NGFS simulations, we have according to NGFS's website [9]: **Net Zero 2050, Divergent Net Zero, Nationally Determined Contributions (NDCs), Current Policies**.

Four sectors [10]:

1. **Very High Emitting:** Agriculture, Forestry and Fishing.
2. **High Emitting:** Electricity, gas, steam and air conditioning supply, Mining and quarrying, Water supply; sewerage, waste management and remediation activities.
3. **Low Emitting:** Construction, Manufacturing, Transportation and storage.
4. **Very Low Emitting:** Accommodation and food service, Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies, Financial and insurance, Information and communication, Professional, scientific and technical; administrative and support service, Public administration, defence, education, human health and social work, Real estate activities.

Taxes

Emissions level	Very High	High	Low	Very Low
Current Policies	0.	4.301	0.459	0.014
NDCs	0.	6.151	0.656	0.02
Net Zero 2050	0.	8.883	0.948	0.03
Divergent Net Zero	0.	19.029	2.031	0.063

Table 1: Average annual carbon tax on households' consumption from each of the sectors between 2020 and 2030 (in %)

Emissions level	Very High	High	Low	Very Low
Current Policies	4.006	1.605	0.413	0.069
NDCs	5.73	2.296	0.591	0.098
Net Zero 2050	8.275	3.315	0.853	0.142
Divergent Net Zero	17.728	7.102	1.827	0.304

Table 2: Average annual carbon tax on firms' production in each of the sector between 2020 and 2030 (in %)

We define in the same way taxes on intermediary inputs ζ^{ji}

Consumption growth

Emissions level	Very High	High	Low	Very Low	Total
NDCs	-0.205	-0.120	-0.089	-0.016	-0.109
Net Zero 2050	-0.583	-0.335	-0.249	-0.046	-0.306
Divergent Net Zero	-1.134	-0.614	-0.458	-0.089	-0.569

Table 3: Average annual consumption growth evolution with respect to the *Current Policies* scenario between 2020 and 2030 (in %)

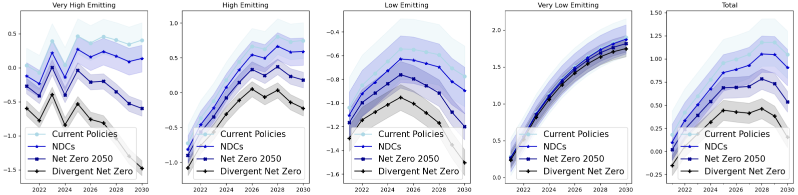


Figure 1: Mean and 95% confidence interval of the annual economic growth from 2020 to 2030

Firm value distribution

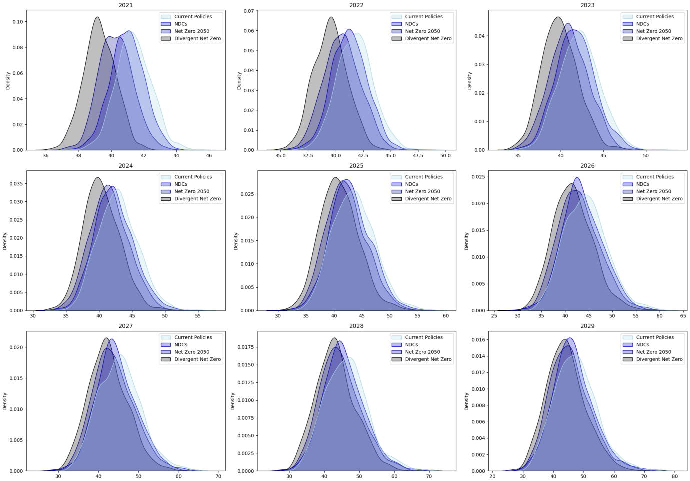


Figure 2: Firm value distribution per scenario and per year

Probability of default

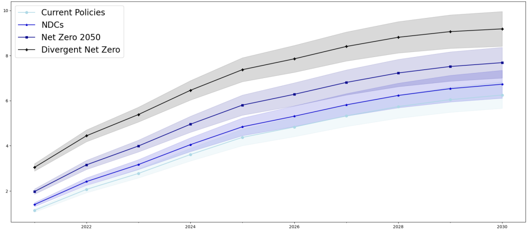
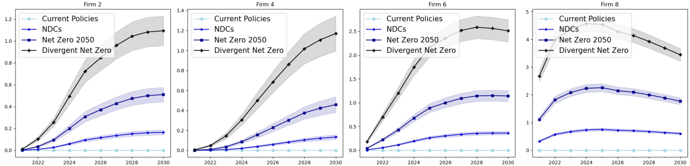


Figure 3: Average annual probability of default per scenario

Over the next 10 years, the annual average PD for the *Current Policies* scenario is 1.963%, for the *NDCs* scenario is 2.179%, for the *Net Zero 2050* scenario is 2.625%, and for the *Divergent Net Zero* scenario is 3.368%.



Expected Loss

Emissions level	Firm 2	Firm 4	Firm 6	Firm 8	Portfolio
Current Policies	0.465	1.069	0.408	4.594	0.933
NDCs	0.562	1.189	0.431	5.069	1.033
Net Zero 2050	0.788	1.433	0.476	6.027	1.238
Divergent Net Zero	1.245	1.818	0.543	7.561	1.577

Table 4: Average annual EL as a percentage of exposure

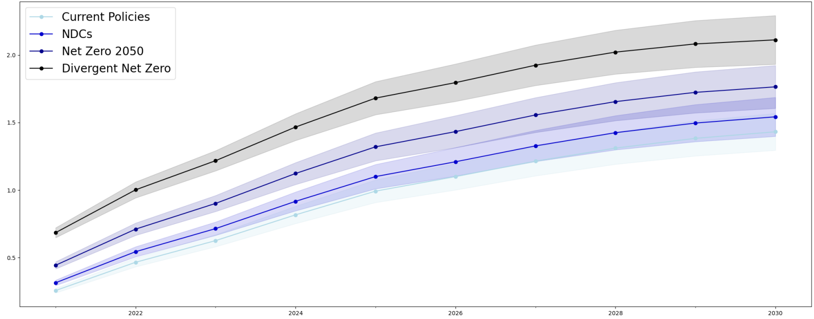


Figure 5: EL of the portfolio in % of the exposure per scenario and per year.

Expected Loss (2)

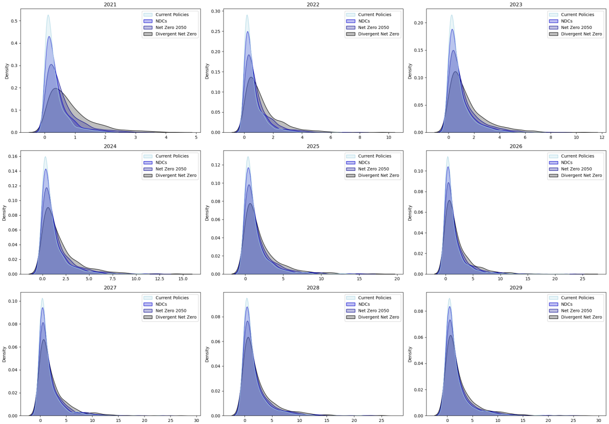


Figure 6: Annual EL distribution per scenario

Unexpected Loss

Emissions level	Firm 2	Firm 4	Firm 6	Firm 8	Portfolio
Current Policies	0.567	0.629	0.133	2.401	0.273
NDCs	0.673	0.689	0.140	2.592	0.297
Net Zero 2050	0.910	0.815	0.153	2.947	0.345
Divergent Net Zero	1.370	1.008	0.173	3.441	0.419

Table 5: Average annual UL as a percentage of exposure

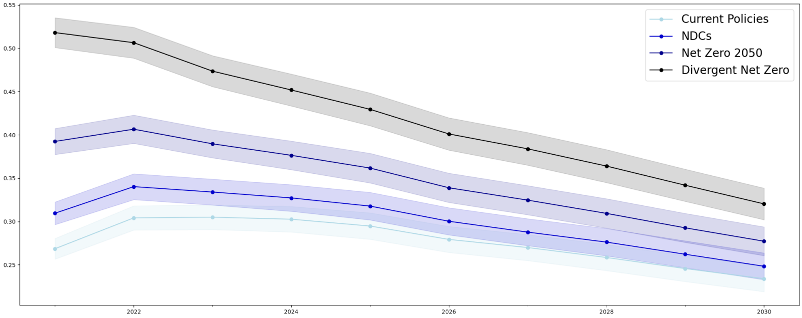


Figure 7: UL of the portfolio in % of the exposure per scenario and per year

Unexpected Loss (2)

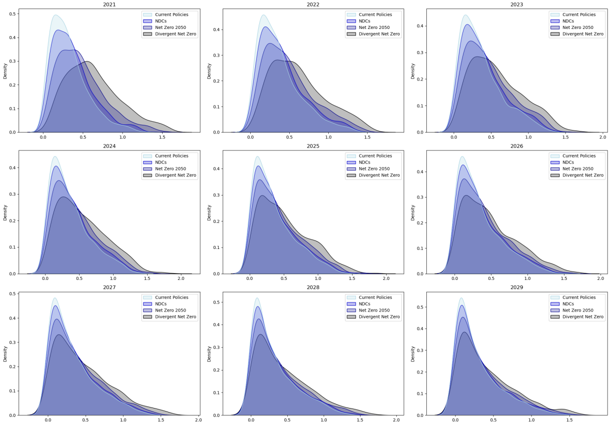


Figure 8: Annual UL distribution per scenario

Losses' sensitivities

Emissions level	**Firm 2**	**Firm 4**	**Firm 6**	**Firm 8**	**Portfolio**
Current Policies	1.561	1.581	1.191	0.827	1.280
NDCs	1.777	1.687	1.261	0.904	1.402
Net Zero 2050	2.142	1.864	1.386	1.035	1.631
Divergent Net Zero	2.668	2.096	1.562	1.215	1.973

Table 6: Average annual EL sensitivity to carbon price in %

Emissions level	**Firm 2**	**Firm 4**	**Firm 6**	**Firm 8**	**Portfolio**
Current Policies	1.299	1.290	1.042	0.547	1.135
NDCs	1.463	1.365	1.102	0.583	1.148
Net Zero 2050	1.726	1.485	1.206	0.634	1.197
Divergent Net Zero	2.070	1.632	1.352	0.681	1.472

Table 7: Average annual UL sensitivity to carbon price in %

For example, over the next ten years, if the price of carbon varies by 1% around the scenario *NDCs*, the EL will vary by 1.402% while the UL will change by 1.148% around this scenario.

Conclusion and future works

This work provides a preliminary methodology to calculate the evolution of credit risk measures of a multisectoral credit portfolio, starting from a climate transition scenario described by a carbon price.

- Exogenous and deterministic scenarios?
- Heterogeneous agents (agent-based or mean field games models)?
- Is LGD deterministic? constant? Independent of the carbon price?
- Does EAD, and thus bank balance sheets remain static over the year?

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